Linear stability of MHD configurations

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Ideal MHD configurations

- Interested in any time-dependent configuration of density, entropy, velocity and magnetic field in $(\rho(\mathbf{r},t), s(\mathbf{r},t), \mathbf{v}(\mathbf{r},t), \mathbf{B}(\mathbf{r},t))$ that obey:
  - $\Rightarrow$ passive entropy advection $(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla)s = 0$
  - $\Rightarrow$ mass conservation $\frac{d}{dt}\rho = -\rho\nabla\cdot\mathbf{v}$
  - $\Rightarrow$ magnetic flux conservation $\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B})$
  - $\Rightarrow$ Equation of motion (EOM), including (self-)gravity

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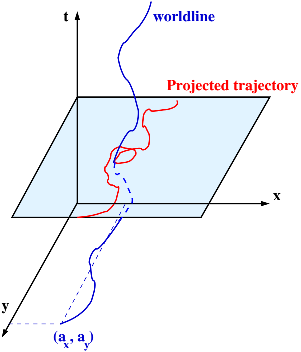
Mappings: trajectories through space-time



- map in fourspace: $(\mathbf{a}, t) \mapsto (\mathbf{r}(\mathbf{a}, t), t)$

  $\Rightarrow$ connects original fluid parcel position to present position

  $\Rightarrow$ at fixed t, projected 3D time-parametrized trajectory has tangent vector

$$\frac{\partial \mathbf{r}}{\partial t}(\mathbf{a}, t) =: \mathbf{v}(\mathbf{a}, t)$$

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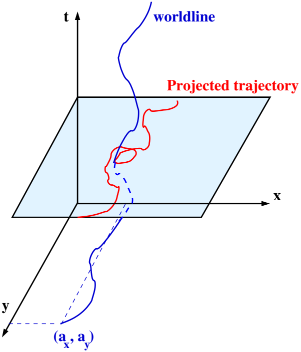

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- Suppose we are given the mapping **a**, together with initial ($t = 0$) density, entropy and magnetic field variation

  $\Rightarrow$ geometric deformation info in tensor $F^i_j = \frac{\partial r^i}{\partial a^j}$

  $\Rightarrow$ its determinant F relates to compression as $\frac{1}{F}\frac{dF}{dt} = \nabla \cdot \mathbf{v}$
  (and incompressible flows treat F like a tracer, as entropy)

- Hence, **formally** we have the full time-evolution of density, entropy and magnetic field at all times when given the mapping, since

$$\Rightarrow \; \rho(\mathbf{a}, t_1) = F^{-1}(\mathbf{a}, t_1)\rho(\mathbf{a}, 0)$$

$$\Rightarrow \; s(\mathbf{a}, t_1) = s(\mathbf{a}, 0)$$

$$\Rightarrow \; B^k(\mathbf{a}, t_1) = F^{-1}(\mathbf{a}, t_1)F_j^k(\mathbf{a}, t_1)B^j(\mathbf{a}, 0)$$

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Energy considerations

- ideal gas with internal specific energy $e^*(T) = \frac{N}{2}\frac{k_B T}{\mu m_p}$ (for N degrees of freedom per particle)

  $\Rightarrow$ thermodynamics rewrites $e^*(s, \rho^{-1})$ [in terms of entropy and specific volume]

- introduce $e = e^*(s(\mathbf{a}), \rho^{-1}) + v_A^2/2$ [Alfvén speed $v_A = \frac{B}{\sqrt{\mu_0 \rho}}$]

  $\Rightarrow$ then energy per unit mass $e(s, \rho^{-1}, B^2)$ obeys

$$\rho(\partial_t + \mathbf{v} \cdot \nabla) e = \bar{\bar{\mathbf{T}}} : \nabla \mathbf{v}$$

where we find the MHD stress tensor

$$\bar{\bar{\mathbf{T}}} = \frac{\mathbf{BB}}{\mu_0} - \left(p + \frac{B^2}{2\mu_0}\right) \hat{\mathbf{I}}$$

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Newton's law

- to obtain EOM, solve stationary action principle
  - $\Rightarrow$ take Lagrangian: $\mathscr{L} = \rho \left(\frac{1}{2} v^2 - \frac{1}{2}\Phi_{int} - \Phi_{ext} - e \right)$
  - $\Rightarrow$ least action principle $\delta \int \int \mathscr{L} \mathrm{d}^3\mathbf{r} dt = 0$
  - $\Rightarrow$ vary over all mappings $\mathbf{r}(\mathbf{a}, t)$, so Euler-Lagrange gives

$$0 = \frac{\partial}{\partial t} \frac{\partial \tilde{\mathscr{L}}}{\partial \frac{\partial r^i}{\partial t}} + \frac{\partial}{\partial a^j} \frac{\partial \tilde{\mathscr{L}}}{\partial \frac{\partial r^i}{\partial a^j}} - \frac{\partial \tilde{\mathscr{L}}}{\partial r^i}.$$

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- this latter findings states that **if we were given all possible mappings**, we would be able to select the one that is physically relevant, namely the one that mimimizes the difference between kinetic and potential energy, since this is the mapping that obey's Newton's law

$\Rightarrow$ we will now use the same idea to link two 'close' solutions which start of with same density, entropy and magnetic field initially, and connect via Lagrangian displacement function $\xi(\mathbf{a}, t) = \mathbf{r}'(\mathbf{a}, t) - \mathbf{r}(\mathbf{a}, t)$

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  $\Rightarrow$ yields following for linearized Eulerian flow quantities

$$\delta \mathbf{v} = \partial_t \boldsymbol{\xi} + \mathbf{v}\cdot\nabla\boldsymbol{\xi} - (\boldsymbol{\xi}\cdot\nabla)\,\mathbf{v}$$

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Linearized field theory

- to get linearized EOM, consider the action that evaluates the Lagrangian density **for the displaced flow**, hence minimize

$$\int\int\left(\frac{v'^2}{2}-\frac{1}{2}\Phi'_{int}-\Phi'_{ext}-e'\right)(\mathbf{r}',t)\rho'(\mathbf{r}',t)\mathrm{d}^3\mathbf{r}'dt$$

$\Rightarrow$ rewrite all terms in powers of ξ (and its spatial derivatives) and use Euler-Lagrange by vary ξ, so compute

$$\frac{\partial}{\partial t}\frac{\partial\mathscr{L}'}{\partial\frac{\partial\xi^i}{\partial t}}+\frac{\partial}{\partial r^j}\frac{\partial\mathscr{L}'}{\partial\frac{\partial\xi^i}{\partial r^j}}-\frac{\partial\mathscr{L}'}{\partial\xi^i}=0$$

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- end result yields

$$
\begin{aligned}
\rho \frac{d^2 \xi}{dt^2} = \rho \mathscr{G}(\boldsymbol{\xi}) \quad \equiv \quad & \nabla \left[\left((\gamma - 1)\, p + \frac{B^2}{2\mu_0} \right) \nabla \cdot \xi \right] \\
& + \nabla \xi \cdot \nabla \left(p + \frac{B^2}{2\mu_0} \right) + \left[p + \frac{B^2}{2\mu_0} \right] \nabla \left(\nabla \cdot \xi \right) \\
& + \frac{1}{\mu_0} \left(\mathbf{B} \cdot \nabla \right) \left[\left(\mathbf{B} \cdot \nabla \right) \xi - \left(\nabla \cdot \xi \right) \mathbf{B} \right] \\
& - \frac{1}{\mu_0} \nabla \left[\mathbf{B} \cdot \left(\left(\mathbf{B} \cdot \nabla \right) \xi \right) \right] \\
& - \rho \left(\boldsymbol{\xi} \cdot \nabla \right) \nabla \left(\Phi_{ext} + \Phi_{int} \right) - \rho \nabla \delta \Phi_{int,\xi}
\end{aligned}
$$

$\Rightarrow$ here $\delta\Phi_{int,\xi}(\mathbf{r}) = -\int \frac{G\delta\rho(\mathbf{x})}{|\mathbf{r}-\mathbf{x}|}\, \mathrm{d}^3\mathbf{x}$ is self-gravity perturbation

- main observations are:

  $\Rightarrow$ operator $\mathscr{G}(\boldsymbol{\xi})$ turns out to be self-adjoint (w.r.t. inner product $\langle \boldsymbol{\eta}, \boldsymbol{\xi} \rangle = \int \rho \, (\boldsymbol{\eta}^* \cdot \boldsymbol{\xi}) \, dV$, and this while it is an operator taken from an arbitrary time-evolving MHD state

  $\Rightarrow$ at every time in a nonlinear MHD evolution: can construct a (different) operator $\mathscr{G}$ which is self-adjoint

  $\Rightarrow$ note that written as previously, $\mathscr{G}$ relates to $(\rho(\mathbf{r}, t), p(\mathbf{r}, t), \mathbf{B}(\mathbf{r}, t))$, no explicit occurence of $\mathbf{v}$

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- operator similar to the one introduced by Frieman and Rotenberg in 1960, for describing perturbation about stationary equilibria, where they wrote

$$\mathbf{G}_{FR}(\xi) - 2\rho\mathbf{v}\cdot\nabla\frac{\partial\xi}{\partial t} - \rho\frac{\partial^2\xi}{\partial t^2} = 0$$

$\Rightarrow$ one can show that

$$\mathscr{G}[\xi] = \frac{\boldsymbol{G}_{FR}[\xi]}{\rho} + (\mathbf{v}\cdot\nabla)^2\xi$$

MHD wave signals

- **homogeneous plasma**: slow, Alfvén, fast waves, stable!
  $\Rightarrow$ the **phase speed diagrams** quantify for every angle ϑ between **k** and **B** how far a plane wave can travel in fixed time
  $\Rightarrow$ how does this modify when self-gravity is included?

- Chandrashekar & Fermi (1953); Strittmatter (1966)
  - $\Rightarrow$ static uniform, magnetized medium, **WITH self-gravity**
  - $\Rightarrow$ adopts **Jeans swindle** (is NOT a real equilibrium ...)
- usual analysis $\exp(i\,(\mathbf{k}\cdot\mathbf{r}-\omega t))$ gives dispersion relation
  - $\Rightarrow$ Alfvén wave $\omega^2 = \omega_A^2 = \frac{(\mathbf{k}\cdot\mathbf{B})^2}{\mu_0\rho}$
  - $\Rightarrow$ slow/fast pair from

$$\omega^4 - \left(v_s^2 + v_A^2\right)\omega^2 k^2 + \frac{(\mathbf{k}\cdot\mathbf{B})^2}{\mu_0\rho} v_s^2 k^2 = 0$$

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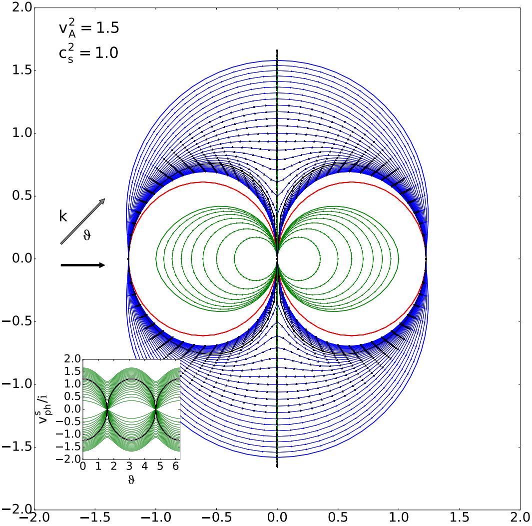
- strong field case



- SELF-GRAVITY, short wavelength below λ_{crit}, i.e. $k > k_{crit}$
- Jeans length: gravity wins from compression

$$\lambda_{crit} = \sqrt{\frac{\gamma p \pi}{G\rho^2}}$$

$\Rightarrow$ **fast phase speed isotropic Alfvén, slow marginal**

- SELF-GRAVITY, wavelength ABOVE λ_{crit}, i.e. $k < k_{crit}$

$\Rightarrow$ unstable slow, fast modes very anisotropic (stable)

$\Rightarrow$ slow less angle dependent growth for wavelengths

$$v_A^2 + v_s^2(k^2) = 0$$

$\Rightarrow$ maximal growth for wavevectors parallel to $\mathbf{B}$

- all details (and many more references) in Phys. of Plasmas 23, 122117 (2016) [plus erratum PoP 24, 029901 (2017)]
  - $\Rightarrow$ work extended from master thesis Thibaut Demaerel