Generating priors for ionosphere tomography with Gaussian Processes

Mutual benefits between atmospheric research and radio science over polar regions - Workshop

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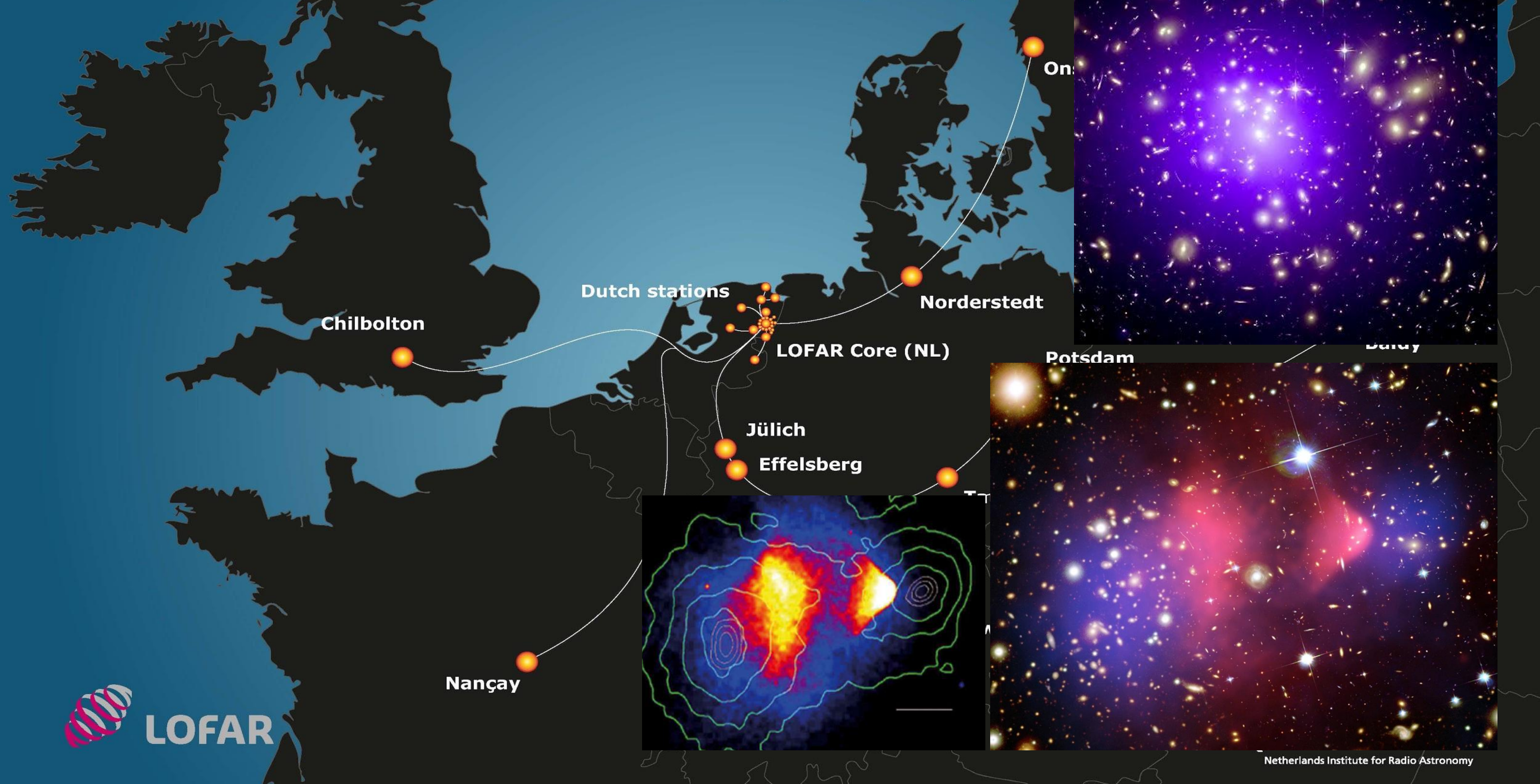
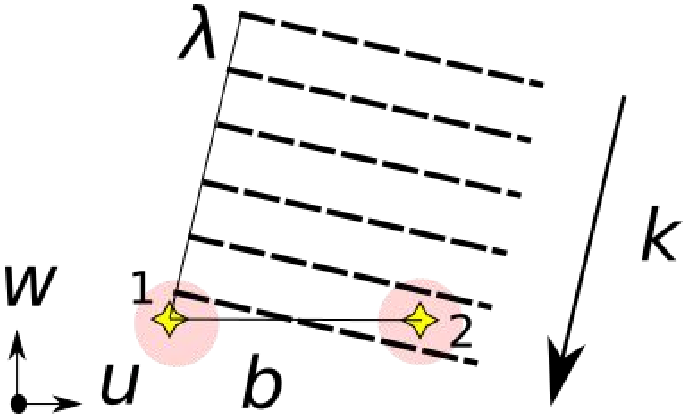
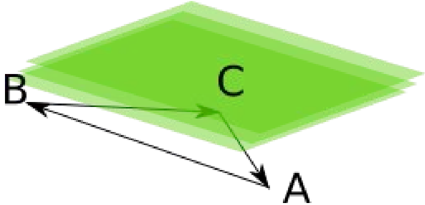


Netherlands Institute for Radio Astronomy

Observables: phase distortions from antenna calibrations

$$\Phi_{A,\alpha} = e^{i\phi_{A,\alpha}} e^{ik\mathbf{x}_A \cdot \mathbf{p}_\alpha}$$





Clock and Propagation terms

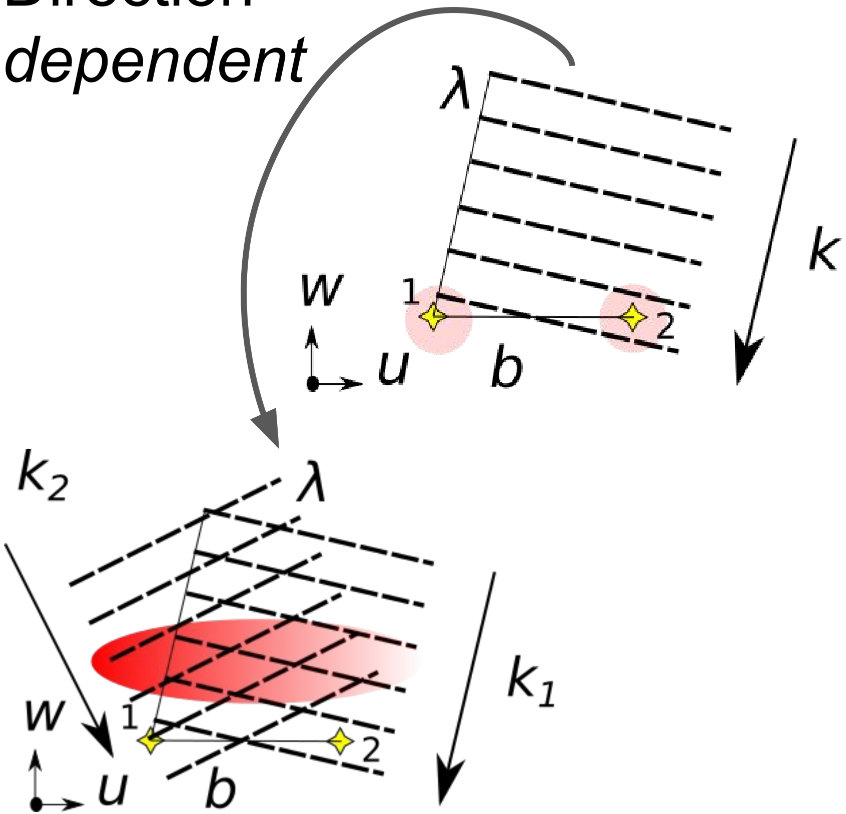
$$\phi_{A,\alpha} \approx 2\pi\nu\tau_A + \frac{C}{\nu}\mathrm{TEC}_{A,\alpha}$$

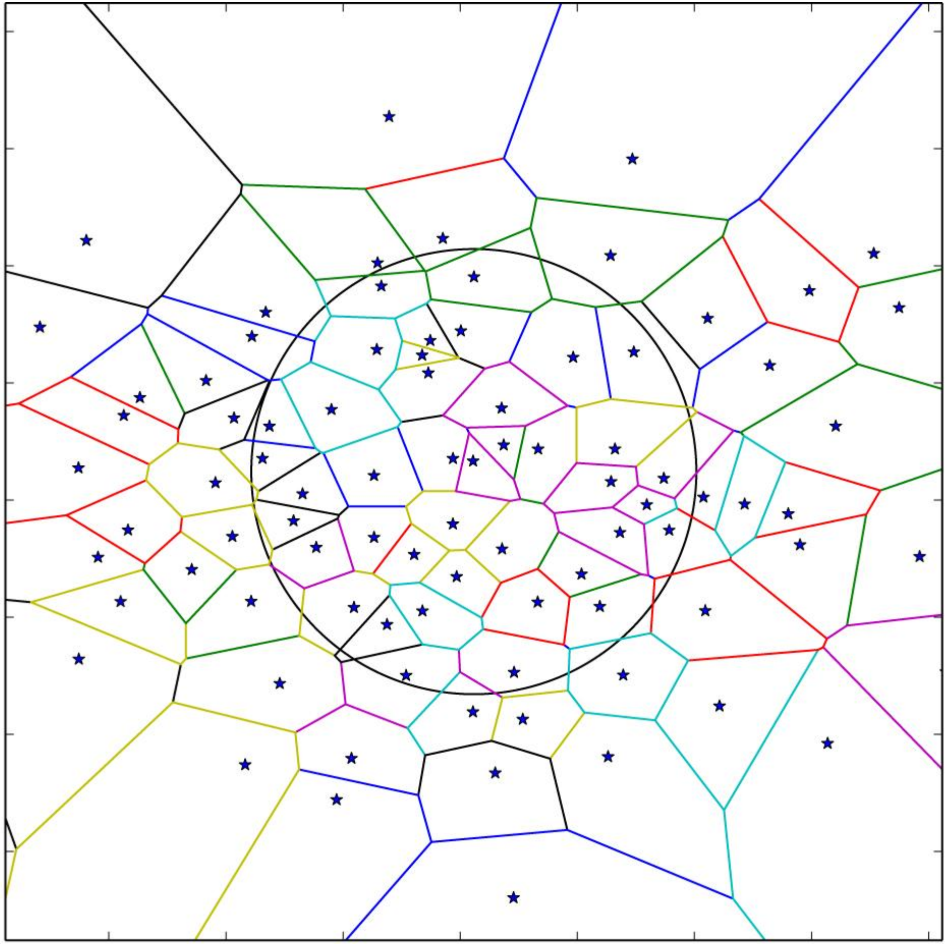
IFF there is **sufficient ν coverage**, and the path differences are less than the *coherence length*, then we can solve...

$$\Delta\mathrm{TEC}_{A,\alpha}$$

Differential electron content (dTEC)

Direction *dependent*





What is a GP?

- In a Bayesian perspective the posterior is the likelihood times the prior normalized by the Bayes evidence
- GP are sampled from a general Gaussian distribution
- Non parametric basis

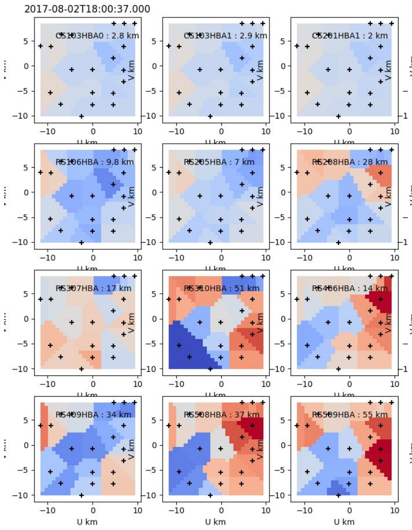
$$P(f_{\mathrm{GP}}|y, X, \theta) = \frac{P(y|f_{\mathrm{GP}})P(f_{\mathrm{GP}}|\theta)}{P(y|X, \theta)}$$

$$P(f_{\mathrm{GP}}|\theta) = \mathcal{GP}(\mu(\theta), K(\theta))$$

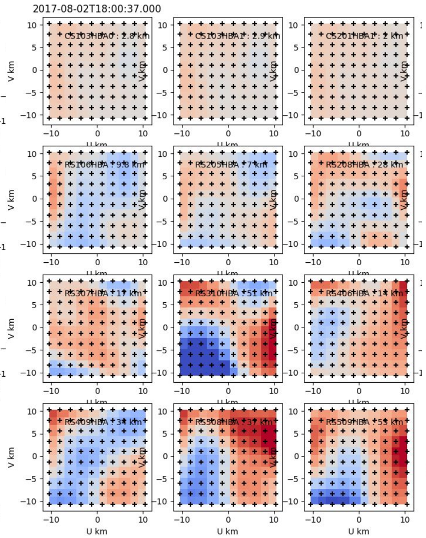
Using Gaussian processes (GP) to model the phase screens

Why GPs?

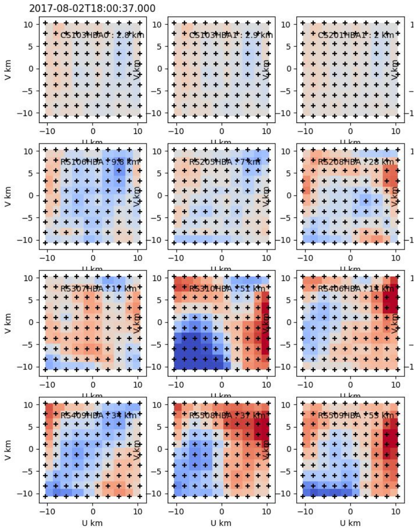
1. Similar models map to similar observables
2. Phase distortions appear (Gaussian?)
3. Nice properties and generalizability
4. Only assumes statistical correlations



Incomplete, noisy observations



Bayesian optimization of a GP



True distribution

$$\phi(t, x, y, \alpha, \beta) \sim \mathcal{G}(0, K(t, x, y, \alpha, \beta))$$

Bayesian modelling the phase screens

$$
\begin{aligned}
K(t,x,y,\alpha,\beta) = \quad & \mathcal{D}(t,x,y,\alpha,\beta) \\
+ \quad & \exp -\frac{t^2}{2\tau_{slow}^2} - \frac{t^2}{2\tau_{quick}^2} \\
+ \quad & \mathrm{RQ}_{1/6}(x,y;l_{inertia}) \exp -\frac{x^2+y^2}{2L_{outer}^2} \\
+ \quad & \exp -\frac{\sin^2(\alpha)+\sin^2(\beta)}{2\gamma^2}
\end{aligned}
$$

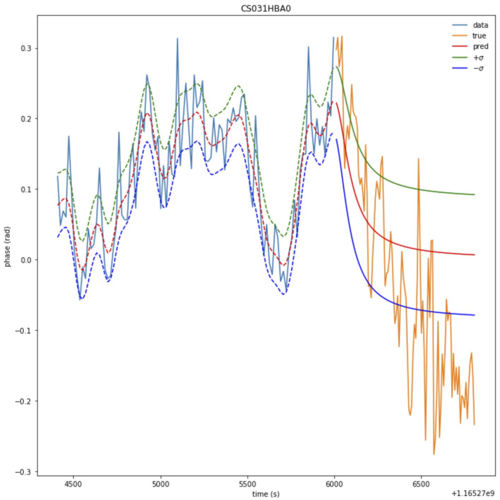
Uncorrelated noise

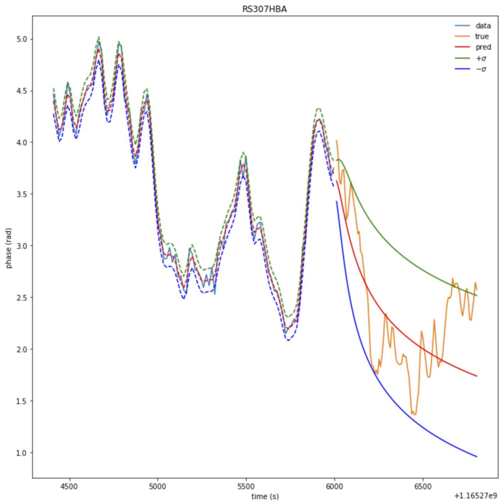
Slow and fast disturbances

Kolmogorov Turbulence

Angular coherency

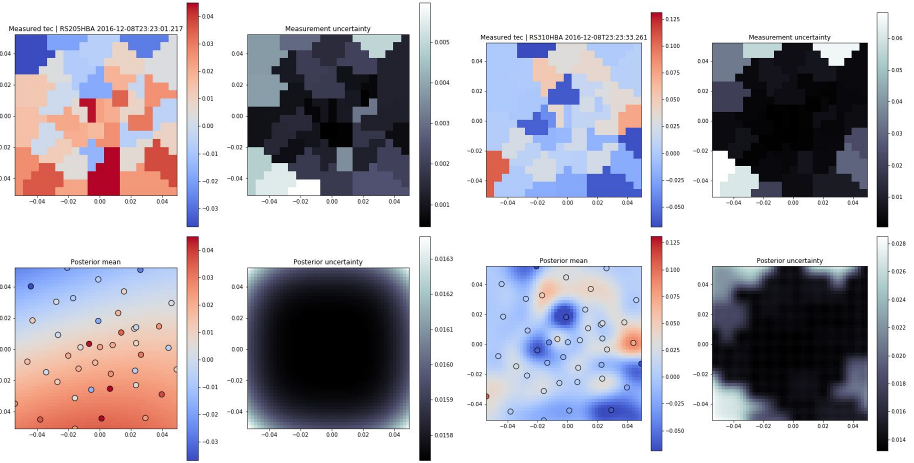
$$
\mathrm{RQ}_\alpha(r;l) \propto \int \underbrace{\left(k^{2(\alpha-1)} \mathrm{e}^{-k^2 l^2}\right)}_{\sim E(k)} \mathrm{e}^{-r^2 k^2/2}
$$

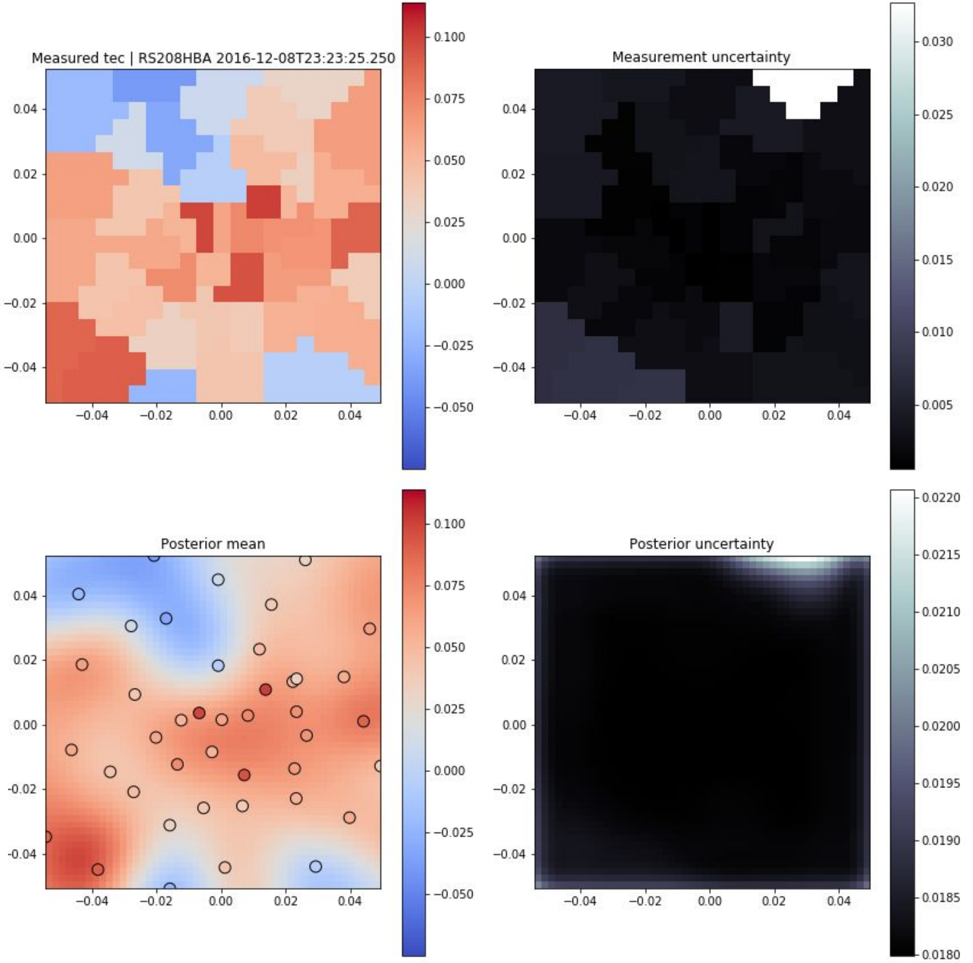




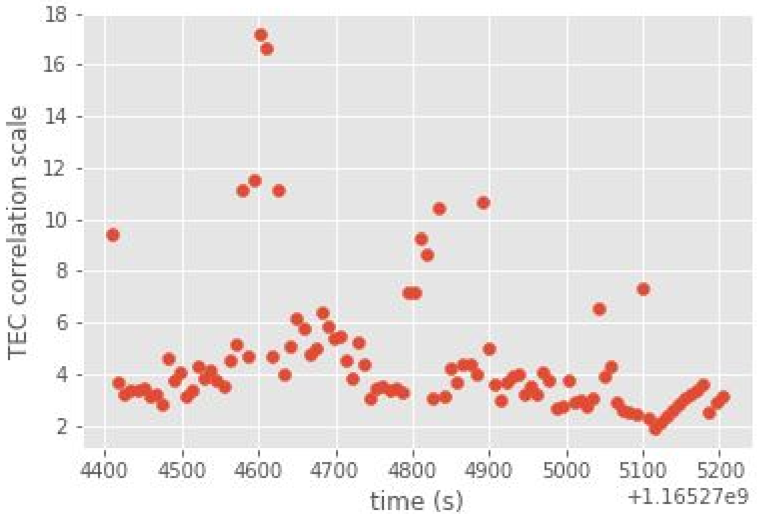
Posterior GP screen

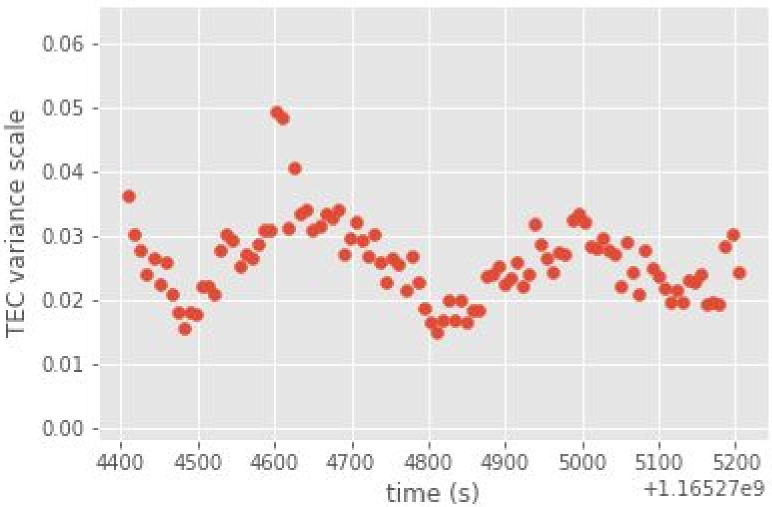
- Manage to smooth noisy data.
- Uncertainty improves from measured.
- Worst case gives a TEC gradient, similar to O(1) polynomial





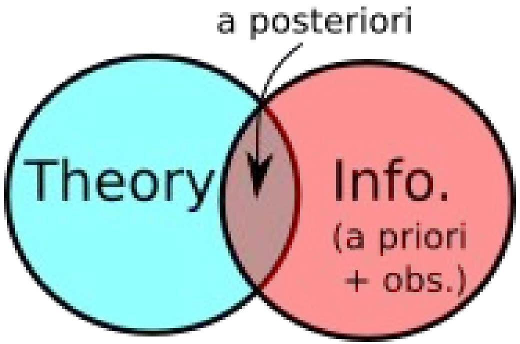
The time series of the GP hyper parameters





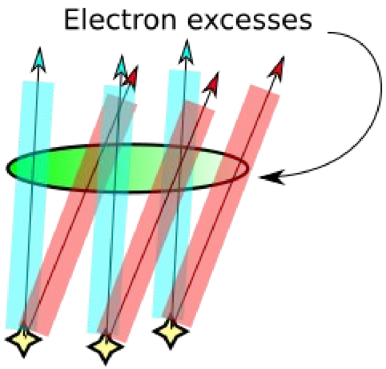
Motivation

Phase screens from 3D Modelling



- $\mu(\mathbf{x})$ is log-electron density
- τ_{ij} are antenna-based slowly varying clock errors
- c_i are antenna based constant phase offset.

$$\mathbf{g}^{ijkl}(\mathbf{m}) = c_i + 2\pi\nu^l\tau_{ij} - \frac{2\pi\nu^l}{c}\left(\int_{\mathcal{R}^{ijk}} ds\, 1 - n(\nu^l,\mu) - \int_{\mathcal{R}^{i_0jk}} ds\, 1 - n(\nu^l,\mu)\right),$$

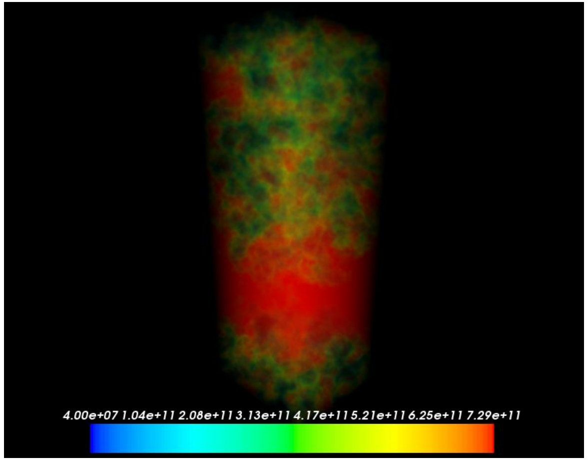


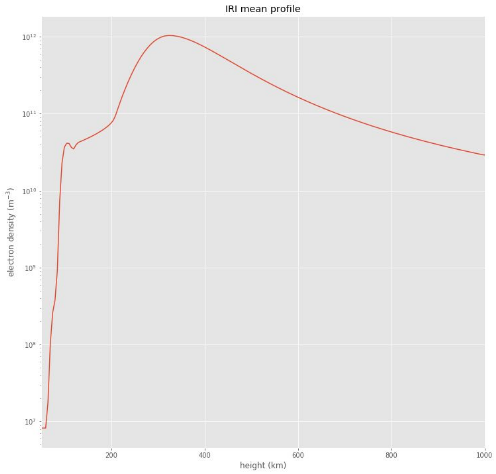
$$n(\nu,\mu) = \sqrt{1 - \frac{K\exp\mu}{n_p(\nu)}}.$$

- Can't do full Bayesian modelling…
- 3000 timesteps x 60 antennas x 40 directions x 125 frequencies
  - Ionosphere ~1e6 voxels
- But we can use the 2D modelling results to help us choose time variable a priori

Linking 2D to 3D

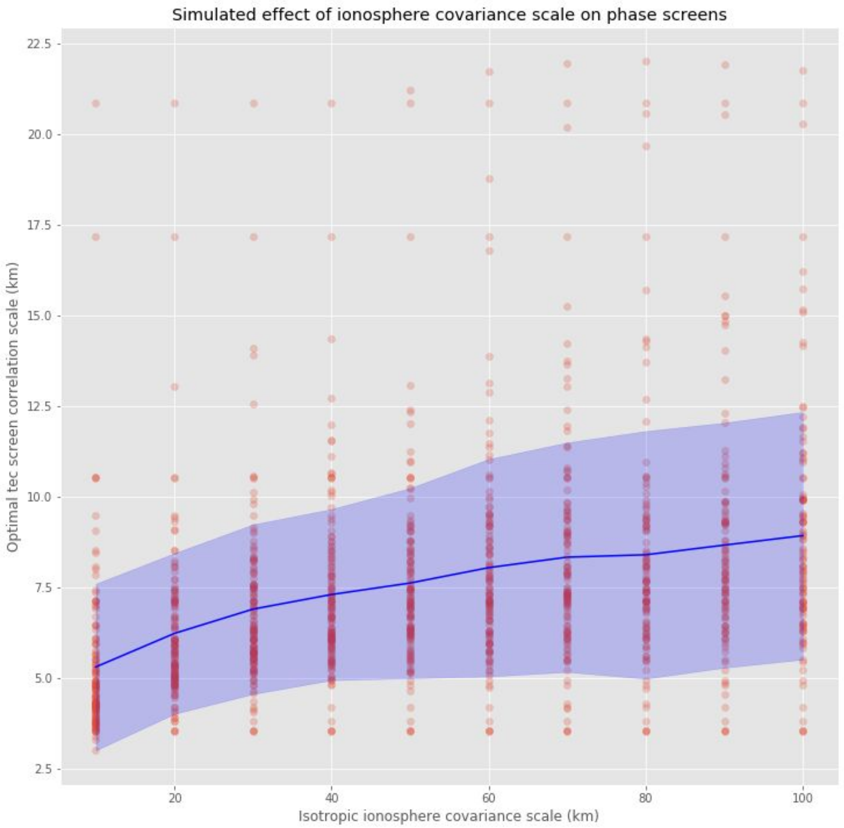
1. We simulate a multitude of unconditioned ionospheres with known properties
   a. IRI mean profile
   b. log-Gaussian random perturbations
   c. We lack a better a priori model most definitely!
2. For covariances, we choose Matern52 with two parameters.
   a. Correlation scale
   b. Variance scale



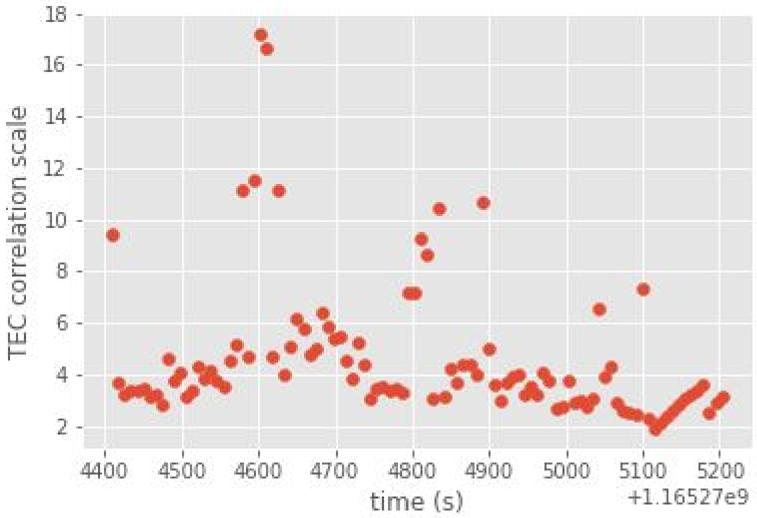


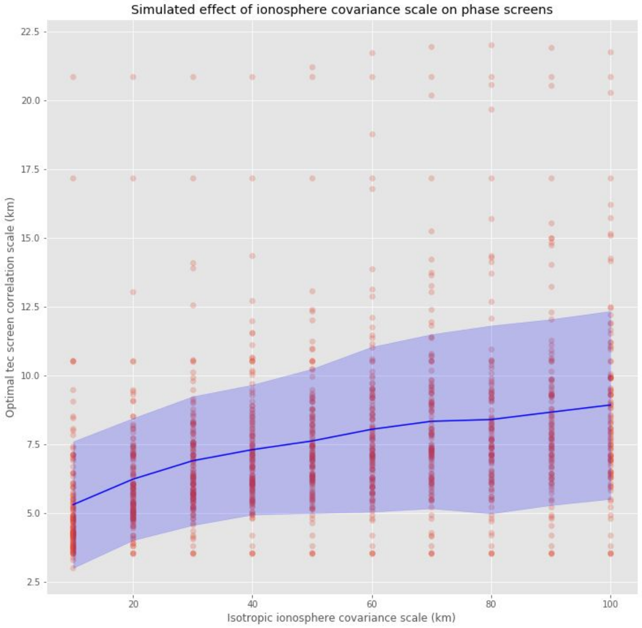
Linking 2D to 3D

1. We simulate a multitude of *unconditioned* ionospheres with known properties
   a. IRI mean profile
   b. log-Gaussian random perturbations
   c. We lack a better a priori model most definitely!
2. Create a *lookup table* linking Bayesian optimal GP TEC-screens to ionosphere properties (blue line).



Connecting data with simulated lookup table.





Conclusions

- GP's offer a nice tool for understanding ionosphere 3D structures
  - Only assumes similar model correlations scales map to similar data correlations scales.
- It seems possible to tie simulated lookup tables to real data.
- Clearly, there is room for collaboration between our communities