Testing a Star Identification Algorithm for a Low-cost In-house Star Tracker

Speaker: Chung-Sheng Lin

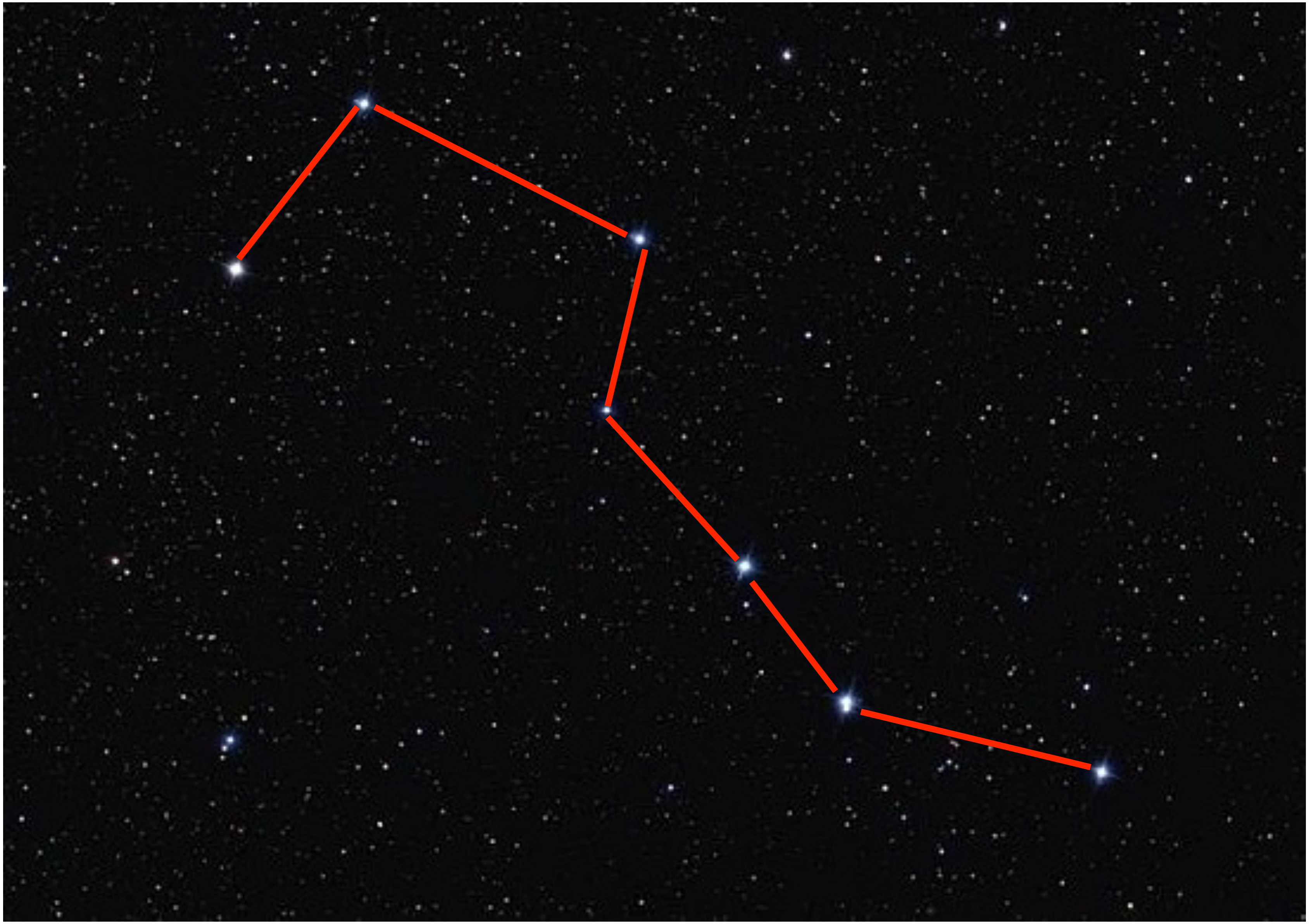
Chung-Sheng Lin and Chi-Kuang Chao

Graduate Institute of Space Science, National Central University, Taiwan

Star trackers can determine the spacecraft attitude by identifying stars. How can the stars be identified?

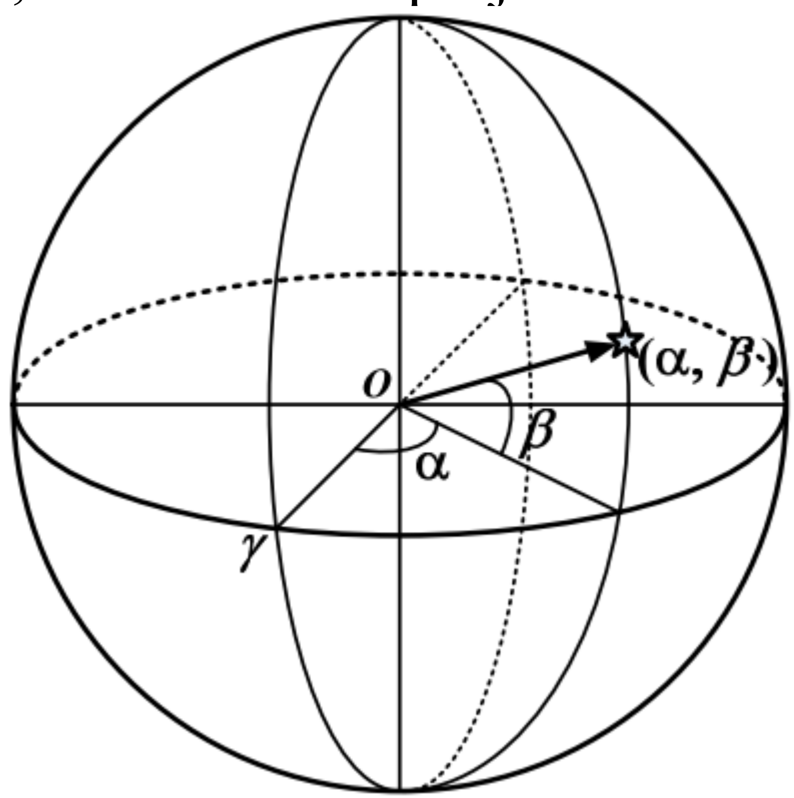
http://pic3.nidbox.net/28/y1323091111_ac55c300_28.jpg

Using the shape, and comparing to known constellations.



Celestial coordinates

Celestial coordinates are often used for a star tracker. The position of the stars on the surface are expressed using right ascension α and declination β, which will be projected onto the star tracker image.

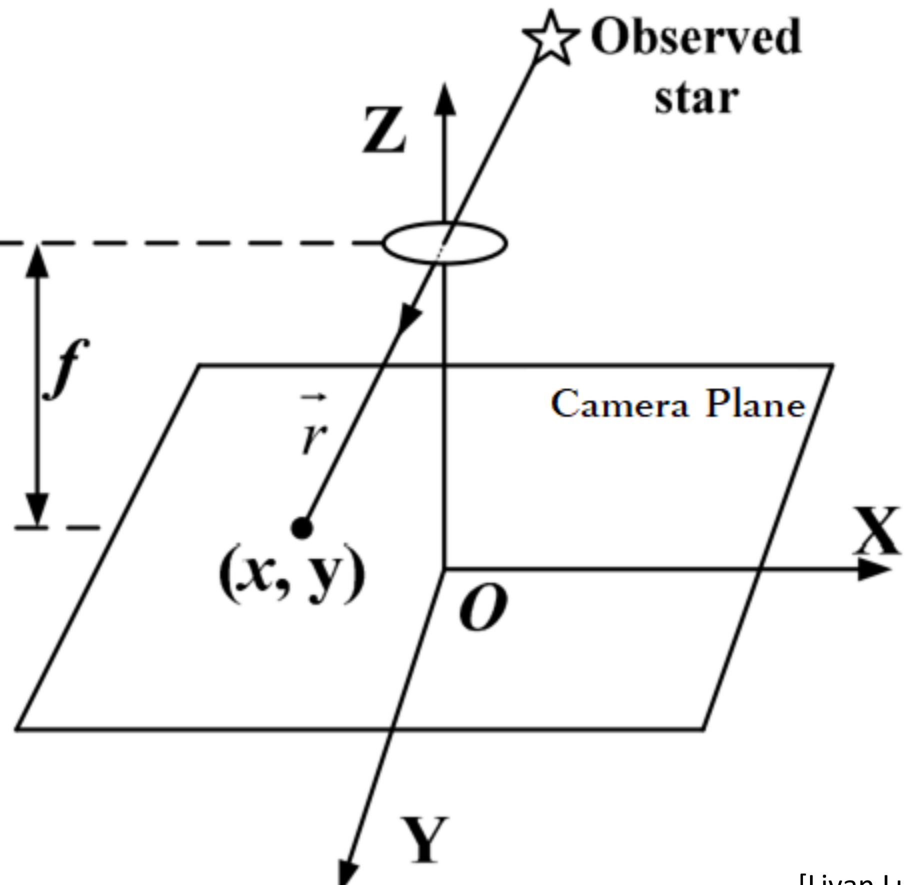


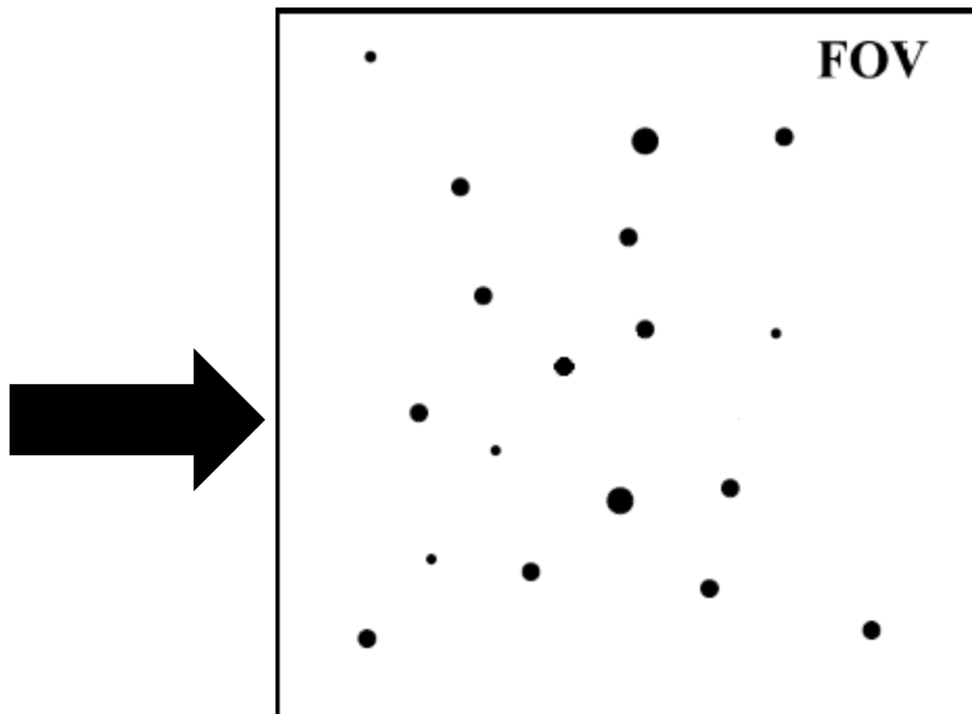
O is origin of the celestial coordinate.
γ is the direction toward the vernal equinox.
α is the right ascension.
β is the declination.

[Liyan Luo et al., 2015]

The image from the star tracker

Coordinates must be defined for the star tracker image. The x axis and the y axis are on the image plane. The z axis is toward the sky, so it is the boresight (a axis of the sight).

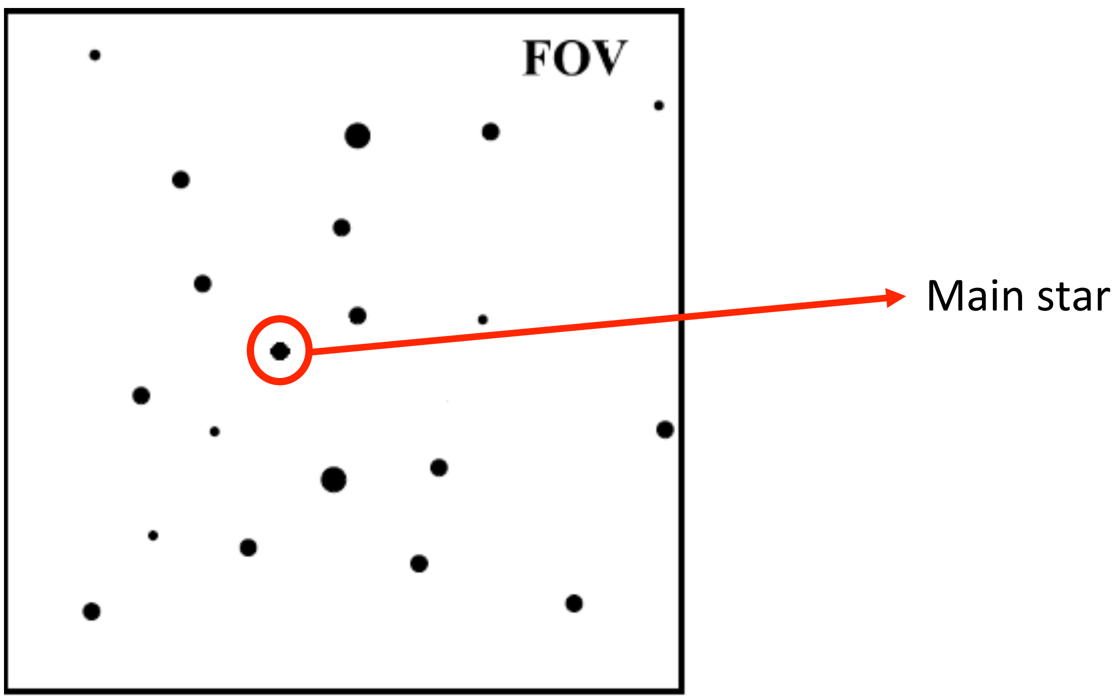




[Liyan Luo et al., 2015]

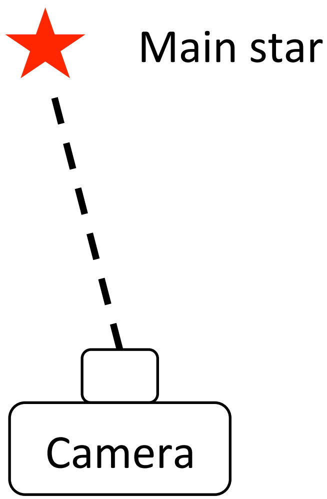
Choosing the main star

We want to identify unique features in the star tracker image. We will choose a main star to be the new center of the image. The characteristics of the unique features will be defined according to the main star, helping us to identify the stars in a database of unique features computed from the star catalog.

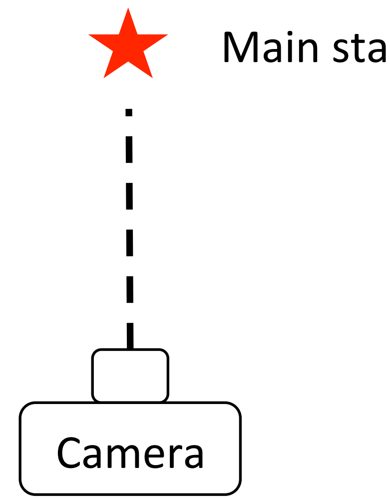


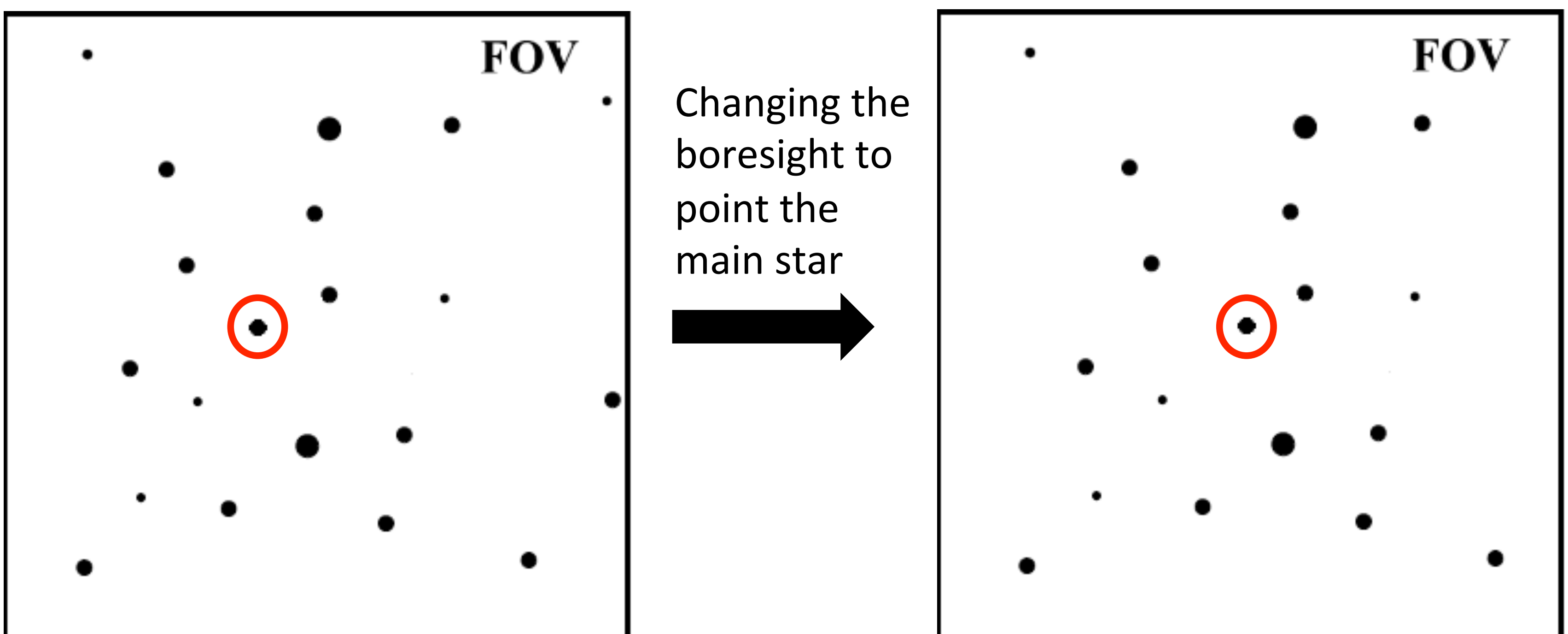
[Liyan Luo et al., 2015]

Centering the main star



There are three methods to center the image boresight on the main star. We can use the Rodrigues' rotation formula method, the projection method and the inverse conversion formula method.





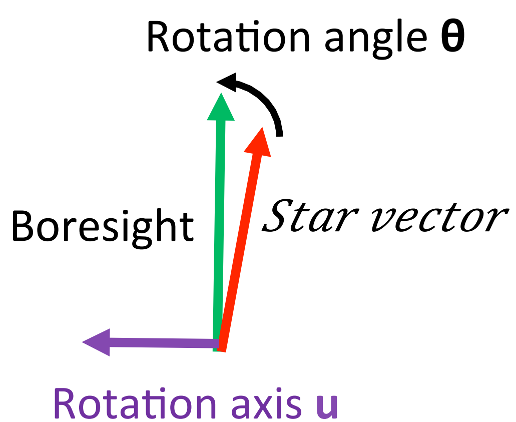
[Liyan Luo et al., 2015]

1. Rodrigues' rotation formula

To convert the star vectors in the image to a new coordinate system centered on the main star, we can follow the Rodrigues' rotation formula , which rotates the rotation axis **u** about the rotation angle **θ** .

$$R_{\mathbf{u}}(\theta) = \cos\theta I + (1-\cos\theta)\mathbf{u}\mathbf{u}^T + \sin\theta\,[\mathbf{u}]_\times$$

$$= \begin{bmatrix} u_1u_1v\theta + c\theta & u_1u_2v\theta - u_3s\theta & u_1u_3v\theta + u_2s\theta \\ u_1u_2v\theta + u_3s\theta & u_2u_2v\theta + c\theta & u_2u_3v\theta - u_1s\theta \\ u_1u_3v\theta - u_2s\theta & u_2u_3v\theta + u_1s\theta & u_3u_3v\theta + c\theta \end{bmatrix}$$



where,

cθ is cosθ

sθ is sinθ

vθ is (1-cosθ)

u is $[u_1 , u_2 , u_3]$

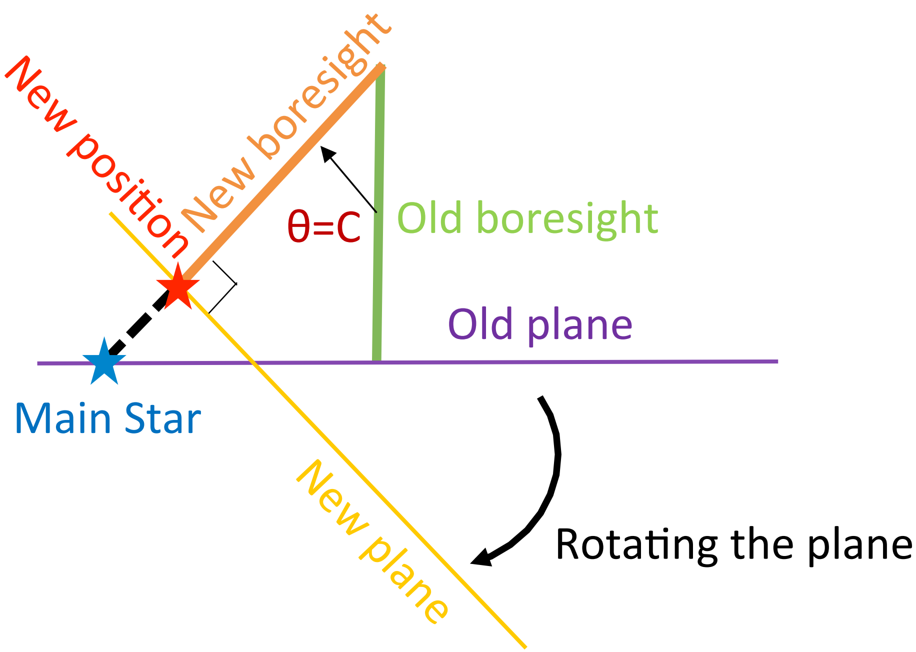
Is this the most efficient method?

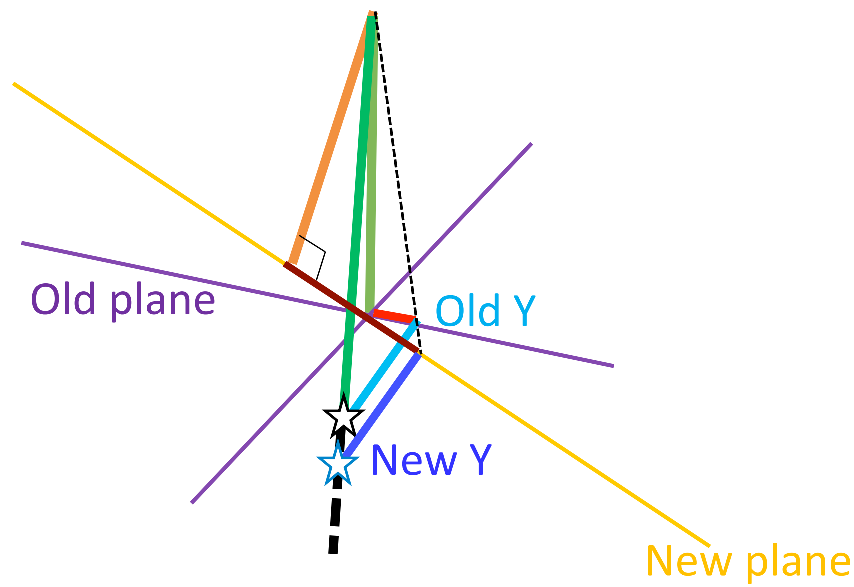
2. Projection: X-position

We can define the new plane, which converts Z-axis to new boresight. The stars project to the plane from the same direction. We can use the tangent and the similar triangles to get the following formula.

$$x' = f\tan(C+a)$$

$$y' = y\sec(C+a)\cos(a)$$





3. Inverse conversion formula

We will use this conversion formula to get the ideal position in the database. It uses the right ascension and the declination of the stars and boresight.

$$x_i = \mathrm{f} \times \cos\delta_i \sin(\alpha_i - \alpha_0) / \sin\delta_i \sin\delta_0 + \cos\delta_i \cos\delta_0 \cos(\alpha_i - \alpha_0)$$

$$y_i = \mathrm{f} \times \sin\delta_i \cos\delta_0 - \cos\delta_i \sin\delta_0 \cos(\alpha_i - \alpha_0) / \sin\delta_i \sin\delta_0 + \cos\delta_i$$

Right ascension of the star:	α_i
Declination of the star:	δ_i
Focal length:	f
Right ascension of the boresight	α_0
Declination of the boresight	δ_0
Position of the star in the image	x_i, y_i

Assumption boresight and conversion formula

We can assume the original boresight points toward the z axis of the celestial coordinates, so the original boresight is toward $\alpha_0 = 0$ and $\delta_0 = 90$.

For each star i:

$$x_i = f \times \frac{\cos\delta_i \sin(\alpha_i - 0)}{\sin\delta_i \sin 90 + \cos\delta_i \cos 0 \cos(\alpha_i - 0)} = \frac{f\cos\delta_i \sin\alpha_i}{\sin\delta_i} = \mathbf{\frac{f\sin\alpha_i}{\tan\delta_i}}$$

$$y_i = f \times \frac{\sin\delta_i \cos 90 - \cos\delta_i \sin 90 \cos(\alpha_i - 0)}{\sin\delta_i \sin 90 + \cos\delta_i \cos 90 \cos(\alpha_i - 0)} = \frac{f\cos\delta_i \cos\alpha_i}{\sin\delta_i} = \mathbf{\frac{f\cos\alpha_i}{\tan\delta_i}}$$

$$\boldsymbol{\alpha_i = \mathrm{atan}\frac{x_i}{y_i}}, \quad \boldsymbol{\delta_i = \mathrm{atan}\frac{f\sin\alpha_i}{x_i}}$$

The result with C code

Testing with the random boresight 10,000 times.

Average position errors (compare with the database, unit is pixels):

```
Rodrigues' rotation:    7.762039e-09
Projection:             9.342598e-11
Conversion formula:     3.370069e-10
```

The position errors of the three methods are small which means that attitude determination errors will be small.

Average time-consumed:

```
Rodrigues' rotation:    8.042193 us
Projection:             0.122227 us
Conversion formula:     6.346633 us
```

The time-consumed are on the order of microseconds, the projection method is almost an order of magnitude faster.

The projection method is the fastest method with the smallest error.

Conclusions

- We can confirm that the three rotation methods have very small error. The projection method is the least time consuming.

- In the future, star catalog identification accuracy will be test in simulations. A star tracker with the tested algorithm will be installed on a hybrid sounding rocket for validation. If validated, we will use it on a future CubeSat mission.